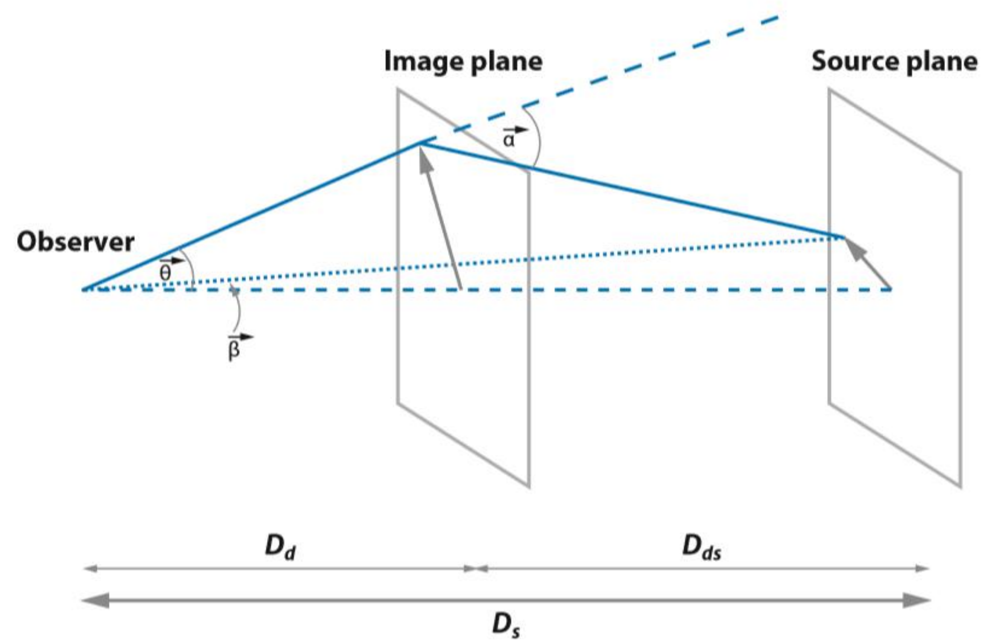
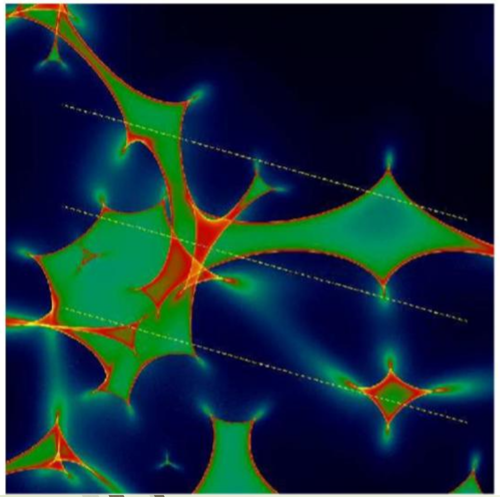
The wave nature of continuous gravitational waves from microlensing

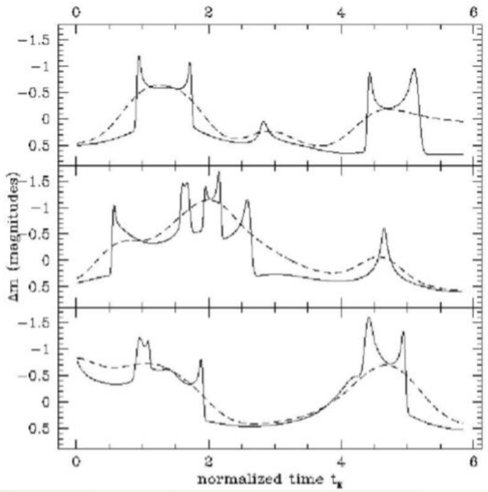
(Kai Liao, Marek Biesiada, Xilong Fan, 2019, ApJ, 875, 139)

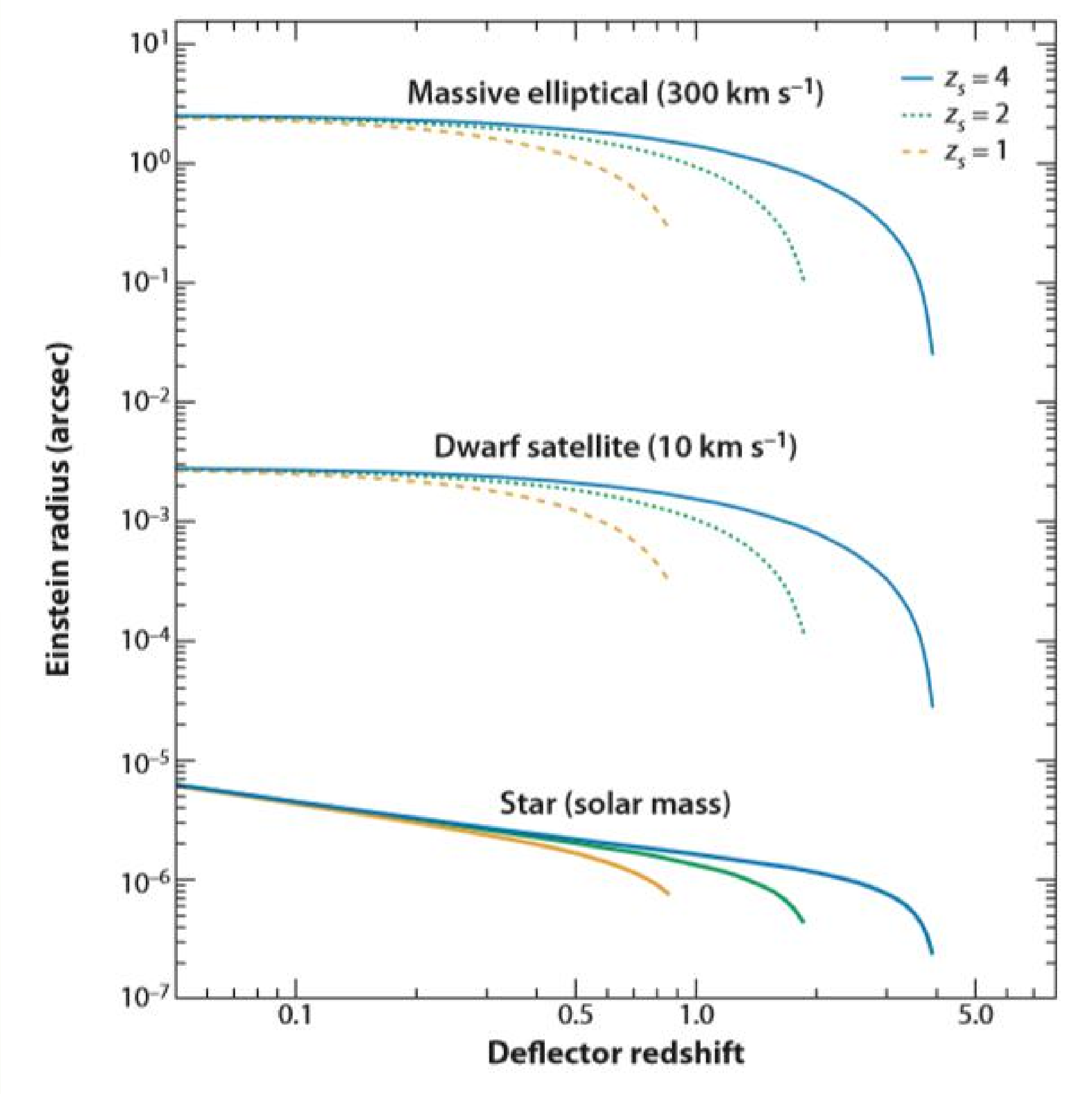
武汉理工大学（WHUT）
廖恺（Kai Liao）

Gravitational Lensing of light





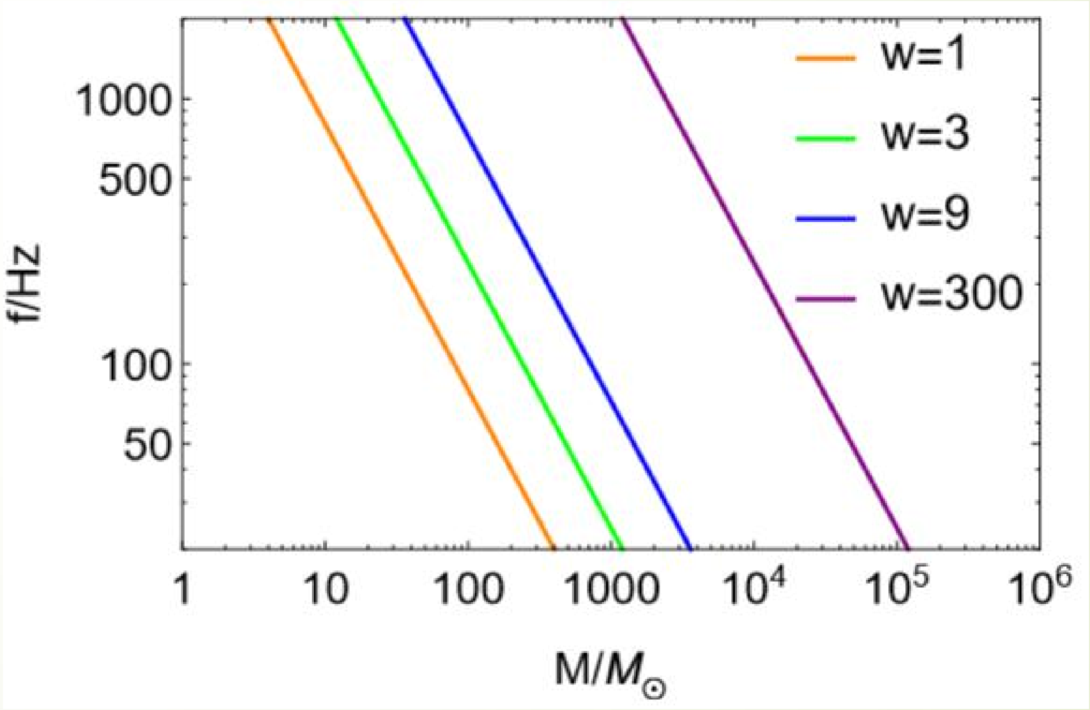




GW lensing:
geometric optics or wave optics

Lens mass (Schwarzschild radius) vs wavelength

$$w = 8\pi M_{Lz} f$$



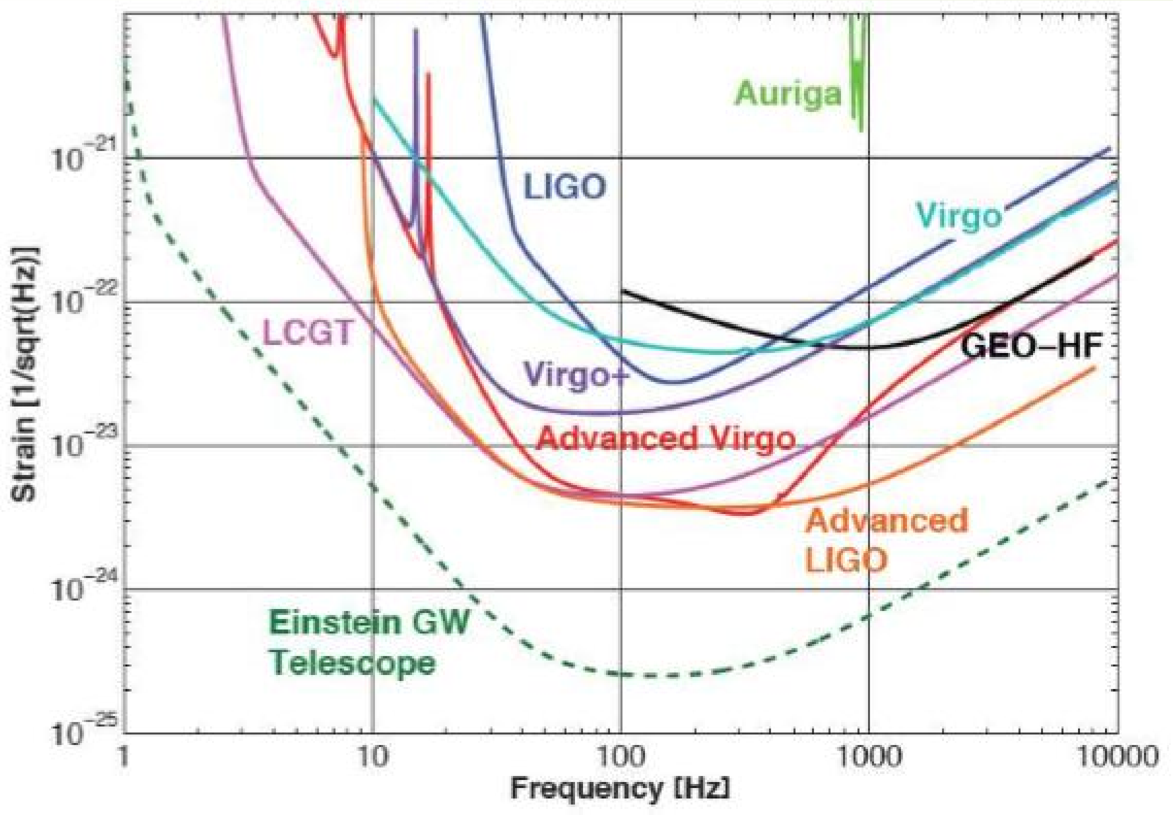
Geometric optics: ET(1-2 Gpc, hundreds thousands of detections per year)

- 50-100 lensed GWs

  (Ding et al. 2015, JCAP)
- Testing the GW speed

  (Fan, Liao et al. 2017, PRL)
- Precision cosmology

  (Liao et al. 2017, Nature Communications)
- Probing the dark matter substructure (Liao et al. 2018, ApJ)



Wave optics description (weak field)

$$g_{\mu\nu} = g^{(L)}_{\mu\nu} + h_{\mu\nu}$$

$$h_{\mu\nu} = \phi e_{\mu\nu}$$

$$\partial_\mu\left(\sqrt{-g^{(L)}}\, g^{(L)\mu\nu}\partial_\nu\phi\right) = 0$$

$$\left(\nabla^2 + \tilde{\omega}^2\right)\tilde{\phi} = 4\tilde{\omega}^2 U \tilde{\phi}$$

Diffraction and interference

(Takahashi & Nakamura 2003, Cao 2014)

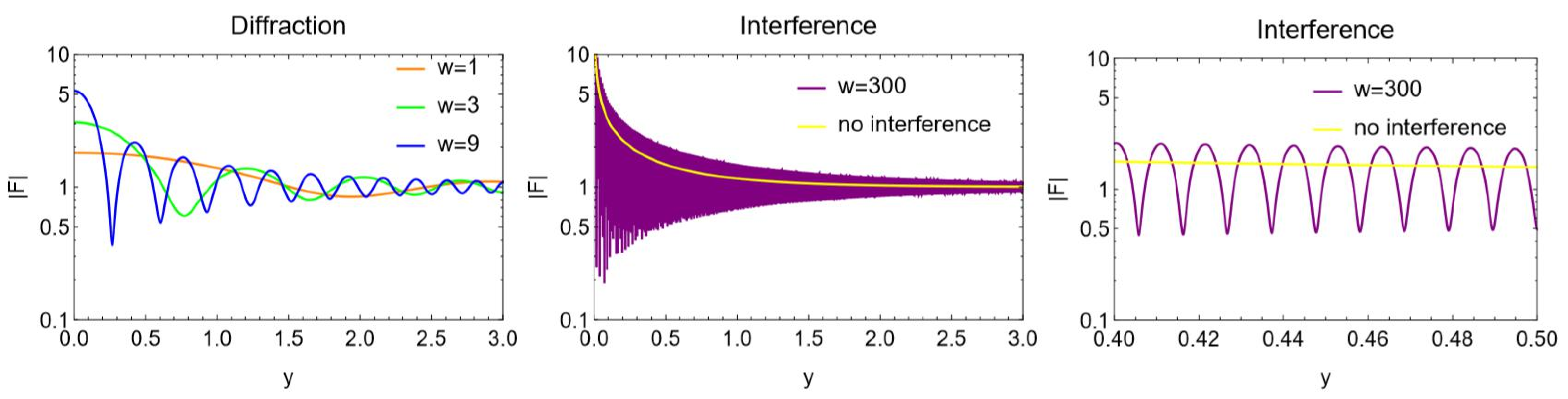
$$F(f) = \tilde{\phi}^L(f)/\tilde{\phi}(f)$$

$$F(f) = \frac{D_s R_E^2 (1+z_l)}{D_l D_{ls}} \frac{f}{i} \int d^2\mathbf{x}\, exp\,[2\pi i f t_d(\mathbf{x},\mathbf{y})]$$

$$t_d(\mathbf{x},\mathbf{y}) = \frac{D_s R_E^2 (1+z_l)}{D_l D_{ls}} \left[\frac{1}{2}|\mathbf{x}-\mathbf{y}|^2 - \psi(\mathbf{x}) + \phi_m(\mathbf{y})\right]$$

$$|F(f)| = \sqrt{|\mu_+| + |\mu_-| + 2|\mu_+\mu_-|^{1/2} \sin(2\pi f \Delta t_d)}$$

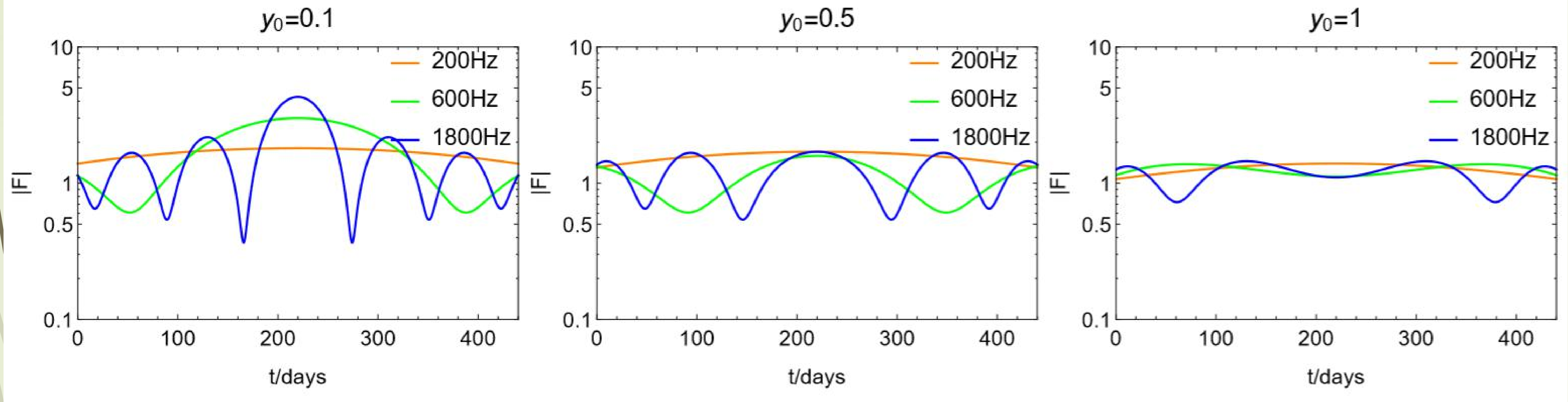
Spatial fringes (point mass lens)

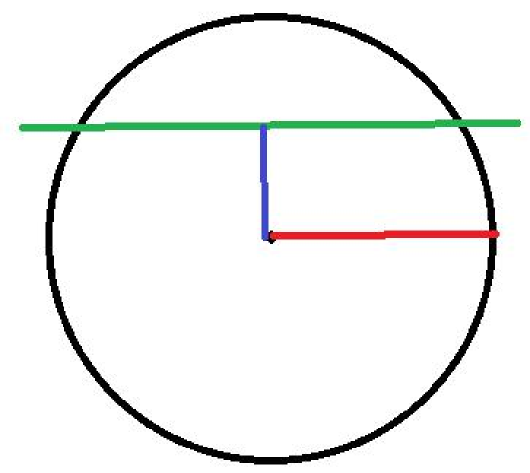


- $w > 1$ to have enough amplification variation;
- $y < 3$ to detect fringes before they are damped;
- $\Delta t > t_f$ **to see a fringe pattern.**

Diffraction modulation

$$\mathbf{v}_{eff} = \mathbf{v}_s - \frac{1+z_s}{1+z_l}\frac{D_s}{D_l}\mathbf{v}_l + \frac{1+z_s}{1+z_l}\frac{D_{ls}}{D_l}\mathbf{v}_{obs}$$

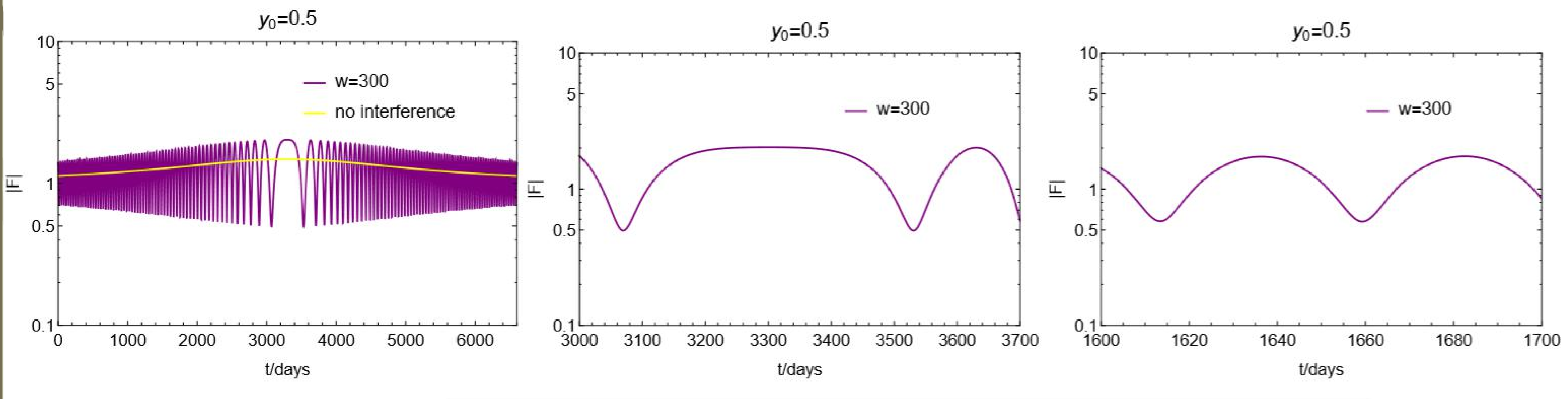




$$t_E \approx 34.7\ day\ \sqrt{4\frac{D_l}{D_s}\left(1-\frac{D_l}{D_s}\right)}\left(\frac{D_s}{8\ kpc}\right)^{1/2} \times \left(\frac{M}{M_\odot}\right)^{1/2}\left(\frac{v_{eff}}{200\ km/s}\right)^{-1},$$

$$t_f = t_E/w$$

Interference modulation



Probability

$$\tau = \frac{1}{\delta\Omega}\int n(D_l)\pi(y_{max}\theta_E)^2 dV$$

$$\tau = \frac{1}{2c^2}\frac{GM(<D_s)}{D_s} = \frac{v_{\rm rot}^2}{2c^2}$$

$$\tau_{\rm GW} \sim f_l\, y_{\rm max}^2 \times 10^{-7} \sim f_l \times 10^{-6}$$

$$P \sim \tau_{\rm GW}\frac{\Delta t}{t_f}$$

$$P \sim f_l(t_f/1\ \text{month})^{-1} \times 10^{-4}$$

$$P \sim f_l\ (1/\phi)(\Delta t/10\ \text{yr})(t_{\rm GW}/1\ \text{month})^{-1} \times 10^{-2}$$

Modulation time scales

Fringe time scales>>motion of the detectors

Last page!   Thank you!